But the values of deflection obtained using the approximate method are very high compared to the values obtained using exact method (some times up to 350% higher). This shows that this approximate method over-estimates the deflection compared to the exact method. This variation is possibly due to the assumed mathematical relationship between the deflection coefficients and the coefficients of the Airy stress function in the approximate method.

4.0 CONCLUSIONS

Large deflection of thin rectangular plates subjected distributed lateral line loads have been mathematically studied by assuming approximate cosine polynomial function in the y direction. Results obtained has been analysed and discussed in comparison with the exact method, leading to the following conclusions;

i. The approximate method generally over-estimates plate behaviour for the same load. However complex it may be, the exact method is more accurate but the approximate method significantly reduces the complexity of the solution albeit with loss of accuracy. The approximate method therefore can be adopted where accuracy is not very critical since it gives higher values.
ii. Although mathematical computer softwares are available to solve load-deflection equations, it is still difficult to get realistic solutions for higher number of coefficients. Good engineering judgment is needed to ascertain accuracy of software output.
iii. The approximate cosine polynomial method does converge fast enough for deflections.

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Exact Large Deflection Analysis of Thin Rectangular Plates under Distributed Lateral Line Load

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ABSTRACT
Research on large deflection of thin rectangular plates to date has focused on plates under uniformly distributed load. There is need to extend the theory to other forms of loads that are encountered by practicing engineers. Exact analysis of large-deflection of thin rectangular plates under distributed lateral line-load is presented in this paper. The analysis is based on solving Von Kármán equations, which relate lateral deflection to applied load and stress. The exact solution is one where the load, deflection, and Airy stress functions are represented by double Fourier series and both x and y coordinate axes are along the plate edges. The functions are substituted into the Von Kármán equations to get third degree polynomials describing relationships between load and deflection coefficients. The resulting polynomials are solved using MATLAB function solver, *fsolve* to get defections caused by different loads. Results are plotted to show the trends of relationships between loads and deflections, and deflections and number of coefficients.

Keywords: Airy Stress; Exact analysis, Large deflection; Line load; Von Kármán equations.

1.0 INTRODUCTION

Rectangular plates have found wide application in the construction industry since the emergence of high strength materials. The high strength of these materials enable thin sections to be used to support large loads even while undergoing large deflections, thus calling for precise methods to analyze their behaviour. Attempts at exact solutions were successfully made by Levy (1941) and Levy and Greenman (1942) but the outcome are complex mathematical procedures based on solution of Von Kármán equations (equations 1a,1b) that are difficult to use in practice and are only justified in few critical cases (Ugural, 1999). Consequently, exact solution of large-deflection of plates for very few cases has been achieved. Solutions for other forms of loads apart from few achieved are still needed.

$$\frac{\partial^4 F}{\partial x^4} + 2\frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{\partial^4 F}{\partial y^4} = E\left[\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right] \tag{1a}$$

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D} + \frac{t}{D}\left(\frac{\partial^2 F}{\partial y^2}\frac{\partial^2 w}{\partial x^2} - 2\frac{\partial^2 F}{\partial x \partial y}\frac{\partial^2 w}{\partial x \partial y} + \frac{\partial^2 F}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right) \tag{1b}$$

w is the deflection of the plate, E is Young's modulus, t is the plate thickness, q is the distributed load acting on plate, and D is the plate flexural rigidity such that:
$D = \frac{Et^3}{12\left(1 - v^2\right)}$, v is Poisson's constant, and F is Airy stress function such that $\frac{\partial^2 F}{\partial y^2} = \sigma_x$, $\frac{\partial^2 F}{\partial x^2} = \sigma_y$, $\frac{\partial^2 F}{\partial x \partial y} = -\tau_{xy}$. σ_x, σ_y are the normal stresses in x and y directions, τ_{xy} is the shear stress in x-y plane.

$$F = -\frac{\overline{p}_x y^2}{2} - \frac{\overline{p}_y x^2}{2} + \sum_{m=0,}^{\infty}\sum_{n=0,}^{\infty} f_{mn} \cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b} \tag{1c}$$

$\overline{p}$ denotes in-plane loads. $\overline{p}$ =0 for lateral loads.

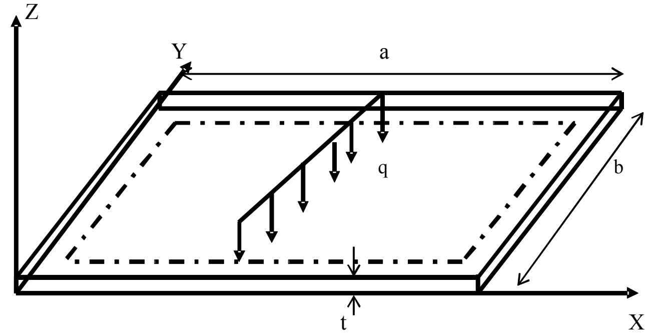

Figure 1: Typical simply supported plate under distributed line load, adapted by authors from literature

2.0 METHODOLOGY

This paper presents the findings of a research effort that analyzed the exact behaviour of thin rectangular plates subjected to uniformly distributed line-loads acting laterally on the plate (Okodi, 2010). The specific objectives were to derive exact load-deflection relationships for thin rectangular plates with simply supported and held edges subjected to distributed lateral line-loads, to solve the derived exact load-deflection relationship for typical simply supported thin rectangular plate. The following steps were followed to achieve the aforementioned objectives:

i) Derivation of exact mathematical relationships between load and deflection coefficients for plates under lateral line-loads,
ii) Solution of the derived exact relationships to obtain the deflections caused by various loads using proprietary software (MATLAB),
iii) Graphical presentation of results.

2.1 Derivations

Consider a simply supported rectangular plate loaded as shown in Figure 2.

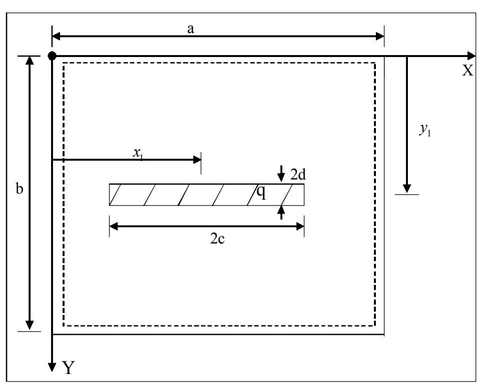

Figure 2: Simply supported thin plate under distributed lateral line load

Load intensity q is given by:

$$q(x, y) = \frac{q}{4cd} \tag{2a}$$

2.2 Assumptions

i) 2d is infinitesimally small compared to other plate dimensions such that its contribution on load distribution is negligible (hence a line load).
ii) 2c is equal to plate length along the x-axis. That is the load is spread on the entire plate length in x direction.

Fourier series representation of lateral load is given by:

$$q(x,y) = \sum_{m=1,2,3..}^{\infty} \sum_{n=1,2,3...}^{\infty} q_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \tag{2b)}$$

q_{mn} are force coefficients; m, n are counters of the series. By Integration applying the assumptions, it can be shown that:

$$q_{mn} = \frac{8q}{\pi mab}\left(\sin\frac{n\pi y_1}{b}\right),$$
$$for\ , m = 1,3,5,7... \tag{2c}$$

This is the equation for determining the load coefficients for uniformly distributed lateral line load.

2.3 Exact large deflection formula for simply supported plate

The general deflected shape formula is represented by expression 3 below.

$$w(x,y) = \sum_{m=1,2,3..}^{\infty} \sum_{n=1,2,3...}^{\infty} w_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \tag{3}$$

w_{mn} ,are deflection coefficients, *m,n* are counters .This satisfies the boundary conditions for the simply supported plate, i.e zero movement at the edges, and no flexural moment along edges $w = 0$ at $x = 0$, a and $y = 0$, b.

$$M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + v\frac{\partial^2 w}{\partial y^2}\right) = 0$$ at x=0,a and

$$M_y = -D\left(\frac{\partial^2 w}{\partial y^2} + v\frac{\partial^2 w}{\partial x^2}\right) = 0$$ at y=0, b.

Substituting the expressions for deflection w and Airy stress function F into the Von Kármán equations and manipulating leads to the following relationship for simply supported plate.

$$\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^2 f_{mn} \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b} =$$
$$\frac{E}{4a^2b^2}\left[\begin{array}{l}\left[r^2s^2w_{rs}^2\left(2\cos\frac{2r\pi x}{a} + 2\cos\frac{2s\pi y}{b}\right) + p^2q^2w_{pq}^2\left(2\cos\frac{2p\pi x}{a} + 2\cos\frac{2q\pi y}{b}\right)\right] \\ + w_{rs}w_{pq}\left\{\begin{array}{l} 2rspq\left[\begin{array}{l}\cos\frac{(r-p)\pi x}{a}\cos\frac{(s-q)\pi y}{b} + \cos\frac{(r-p)\pi x}{a}\cos\frac{(s+q)\pi y}{b} + \\ \cos\frac{(r+p)\pi x}{a}\cos\frac{(s-q)\pi y}{b} + \cos\frac{(r+p)\pi x}{a}\cos\frac{(s+q)\pi y}{b}\end{array}\right] \\ -\left(r^2q^2 + p^2s^2\right)\left[\begin{array}{l}\cos\frac{(r-p)\pi x}{a}\cos\frac{(s-q)\pi y}{b} - \cos\frac{(r-p)\pi x}{a}\cos\frac{(s+q)\pi y}{b} \\ -\cos\frac{(r+p)\pi x}{a}\cos\frac{(s-q)\pi y}{b} + \cos\frac{(r+p)\pi x}{a}\cos\frac{(s+q)\pi y}{b}\end{array}\right]\end{array}\right\}\end{array}\right] \tag{4a}$$

Comparison of terms on the right hands side with those on the left hand side leads to the conclusion that f_{mn} is given by:

$$f_{mn} = \frac{E}{4\left(\frac{m^2 b}{a} + \frac{n^2 a}{b}\right)^2} \sum b_{rspq} w_{rs} w_{pq} \tag{4b}$$

where the summation includes all products for which r+p=m, r-p = $\pm m$ and s+q = n, s-q $=\pm n$ except when r=p=$\frac{m}{2}$ and s=q $=\frac{n}{2}$. Coefficients b_{rspq} are given by: b_{rspq}=2rspq if r=p and s=q. b_{rspq}=2rspq$\pm\left(r^2q^2+p^2s^2\right)$ if $r \neq p, or\ s \neq q$. The sign in this equation is positive when r+p=m and s-q $=\pm n$ or r-p = $\pm m$ and s+q = n, and is negative in all other cases.

Similarly, substituting the expressions for load q, deflection w and Airy stress function F into the Von Kármán equations and manipulating leads to the following relationship for the simply supported plate.

$$q_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} = D\pi^4\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2 w_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} -$$

$$\frac{\pi^4 t}{4a^2b^2}f_{rs}w_{pq}\left[\begin{array}{l}(rq-sp)^2\left[\sin(r+p)\frac{\pi x}{a}\sin(s+q)\frac{\pi y}{b}+\sin(r-p)\frac{\pi x}{a}\sin(s-q)\frac{\pi y}{b}\right] \\ -(rq+sp)^2\left[\sin(r+p)\frac{\pi x}{a}\sin(s-q)\frac{\pi y}{b}+\sin(r-p)\frac{\pi x}{a}\sin(s+q)\frac{\pi y}{b}\right]\end{array}\right] \tag{5a}$$

Considering the similarity of terms on the left and right hand sides of the equation, we simplify the above expression as below.

$$q_{mn} = Dw_{mn}\pi^4\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2 + \frac{t\pi^4}{4a^2b^2}\sum C_{rspq}f_{rs}w_{pq} \tag{5b}$$

where the summation includes all products for which r+p=m, r-p = $\pm m$ and s+q = n, s-q $=\pm n$. Coefficients C_{rspq} are given by: $C_{rspq}=\pm\left(rq \pm sp\right)^2$ if $r \neq 0, and\ s \neq 0$, and $C_{rspq}=\pm\, 2\left(rq\ \pm\ sp\right)^2$ in all other cases. The first sign in this equation is positive when r-p=m or s-q $=n$ (but not at the same time), and is negative otherwise. The second sign is positive if r+p =m and q-s $=\pm n$, or if s+q = n and p-r $=\pm m$ and is negative in all other cases.

Substituting equation 2c into equation 5b for lateral line load, results in the equation 5c below.

$$q = \frac{mab\,\pi^5}{32\sin\frac{n\pi y_1}{b}}\left[4Dw_{mn}\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2 + \frac{t}{a^2b^2}\sum C_{rspq}f_{rs}w_{pq}\right] \tag{5c}$$

This equation presents a direct relationship between the load q and the deflection coefficients. Coefficients f_{rs} in the equation 5c are obtained as presented in equation 4b.

For a case of one deflection coefficient, the load-deflection relationship for simply supported square plate undergoing large deflection would be obtained by substituting w_{11}, *m=n=1, a=b* into equation 5c to get the expression below.

$$q = \frac{\pi^5}{32a^2\sin\frac{\pi y_1}{a}}\left[16Dw_{11} + t\sum C_{rspq}f_{rs}w_{pq}\right] \tag{5d}$$

Coefficient C_{rspq} for various combinations of Airy stress and deflection coefficients are computed and shown in table 1 below.

Table 1: Combinations of Airy stress and deflection coefficients

S/No.	$f_{rs}w_{pq}$	C_{rspq}	$C_{rspq}\ f_{rs}w_{pq}$
1	$f_{0,2}\ w_{1,1}$	+8	$8f_{0,2}w_{11} = \frac{E}{4}\left[w_{11}^3\right]$
2	$f_{2,0}\ w_{1,1}$	+8	$8f_{2,0}w_{11} = \frac{E}{4}\left[w_{11}^3\right]$
3	$f_{2,2}\ w_{1,1}$	0	

Substitute for $C_{rspq}\ f_{rs}w_{pq}$ and collect like terms;

$$q = \frac{\pi^5}{32a^2 \sin\frac{\pi y_1}{a}}\left[16Dw_{11} + Et\ 0.5w_{11}^3\ \right] \tag{5e}$$

This is the load-deflection relationship that is used to determine the deflection coefficients for any value of load. Considering only one coefficient of deflection, the deflection at the centre is the equal to the coefficient. This expression has been solved by computer software (MATLAB) for *1mx1m* steel of *1mm* thickness. Similarly the load-deflection equations for 3, 5, 7 and 9 deflection coefficients have been derived and solved.

2.4 Exact large deflection formula for held plates

Held plates are prevented from both in-plane and out-of-plane movement at the edge supports although rotation about the supports is permitted. This condition introduces additional restraints to the case of simply supported edges since the in-pane displacement is prevented along the edges (Timoshenko and Woinowsky-Krieger, 1959). This additional restraint is catered for by introducing the following displacement expressions.

$$u = \int\left[\varepsilon_x^{/} - \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right]dx \quad \text{for x=0 and x=a,} \tag{6a}$$

$$v = \int\left[\varepsilon_y^{/} - \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2\right]dy \quad \text{for y=0 and y=b} \tag{6b}$$

The in-plane displacement in the x-direction along the edges $x=0$ and $x=a$ is:

$$u = \int\left[\varepsilon_x^{/} - \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right]dx = -\frac{\overline{p_x}}{E}a + \upsilon\frac{\overline{p_y}}{E}a - \frac{\pi^2}{8a}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} m^2 w^2{}_{mn}$$

And the displacement in the y-direction along the edges y=0 and y = b is:

$$v = \int\left[\varepsilon_y^{/} - \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2\right]dy = -\frac{\overline{p_y}}{E}b + \upsilon\frac{\overline{p_x}}{E}b - \frac{\pi^2}{8b}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} n^2 w^2{}_{mn}$$

By setting the edge displacement in both directions to zero, it follows that:

$$-\frac{\overline{p_x}}{E}a + \upsilon\frac{\overline{p_y}}{E}a - \frac{\pi^2}{8a}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} m^2 w^2{}_{mn} = 0 \quad \text{and} \quad -\frac{\overline{p_y}}{E}b + \upsilon\frac{\overline{p_x}}{E}b - \frac{\pi^2}{8b}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} n^2 w^2{}_{mn} = 0$$

From which:

$$\overline{p_x} = -\frac{E}{1-\upsilon^2}\left(\frac{\pi^2}{8b^2}\sum_{m,n}^{\infty} n^2 w_{mn}^2 + \upsilon\frac{\pi^2}{8a^2}\sum_{m,n}^{\infty} m^2 w_{mn}^2\right)$$

$$\overline{p_y} = -\frac{E}{1-\upsilon^2}\left(\frac{\pi^2}{8a^2}\sum_{m,n}^{\infty} m^2 w_{mn}^2 + \upsilon\frac{\pi^2}{8b^2}\sum_{m,n}^{\infty} n^2 w_{mn}^2\right)$$

The values of $\overline{p_x}$ and $\overline{p_y}$ are now introduced into the expanded Airy stress function (equation 1c) to get expression for F shown below.

$$F = \frac{Ey^2}{2\left(1-\upsilon^2\right)}\left(\frac{\pi^2}{8b^2}\sum_{m,n}^{\infty} n^2 w_{mn}^2 + \upsilon\frac{\pi^2}{8a^2}\sum_{m,n}^{\infty} m^2 w_{mn}^2\right) + \frac{Ex^2}{2\left(1-\upsilon^2\right)}\left(\frac{\pi^2}{8a^2}\sum_{m,n}^{\infty} m^2 w_{mn}^2 + \upsilon\frac{\pi^2}{8b^2}\sum_{m,n}^{\infty} n^2 w_{mn}^2\right) + \sum_{m=0..}^{\infty}\sum_{n=0...}^{\infty} f_{mn}\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b} \quad (7)$$

Using the same procedure, the following relationship between load and deflection coefficients is derived for held plates.

$$\frac{q_{mn}a^2b^2}{Et^4} = \frac{\pi^4}{12\left(1-\upsilon^2\right)}\frac{w_{mn}}{t}\left(m^2\frac{b}{a}+n^2\frac{a}{b}\right)^2 + \frac{\pi^4}{4}\sum C_{rspq}\frac{f_{rs}}{Et^2}\frac{w_{pq}}{t} + \frac{\pi^4}{8\left(1-\upsilon^2\right)}\frac{w_{mn}}{t}\left[\left(m^2+\upsilon n^2\frac{a^2}{b^2}\right)\sum_{m,n}^{\infty} n^2\left(\frac{w_{mn}}{t}\right)^2 + \left(\upsilon m^2\frac{b^2}{a^2}+n^2\right)\sum_{m,n}^{\infty} m^2\left(\frac{w_{mn}}{t}\right)^2\right] \quad (8a)$$

By substituting the expression for q_{mn} for lateral line load (equation 2c) into equation 8a, the relationship between load and deflection coefficients is established for held plates.

$$\frac{qab}{Et^4} = \frac{mn\pi^5}{32\sin\frac{n\pi y_1}{b}}\left[\frac{w_{mn}}{3t\left(1-\upsilon^2\right)}\left(m^2\frac{b}{a}+n^2\frac{a}{b}\right)^2 + \sum C_{rspq}\frac{f_{rs}w_{pq}}{Et^3}\right] + \frac{mn\pi^5}{64\left(1-\upsilon^2\right)\sin\frac{n\pi y_1}{b}}\frac{w_{mn}}{t^3}\left[\left(m^2+\upsilon n^2\frac{a^2}{b^2}\right)\sum_{m,n}^{\infty} n^2 w_{mn}^2 + \left(\upsilon m^2\frac{b^2}{a^2}+n^2\right)\sum_{m,n}^{\infty} m^2 w_{mn}^2\right] \quad (8b)$$

3.0 RESULTS

The result for 1mx1m simply supported steel plate of 1mm thickness is presented in Table2 below.

Table 2: Showing exact deflections for various numbers of terms

Load	Deflection w(mm) for different numbers of terms				
(N)	1 term	3 terms	5 terms	7 terms	9 terms
0	0	0	0	0	0
1	0.3343	0.3248	0.3253	0.3262	0.3289
3	0.8397	0.8119	0.813	0.8161	0.8351
5	1.1784	1.1362	1.1371	1.1432	1.184
7	1.4314	1.3772	1.3782	1.3871	1.4512
9	1.6354	1.5705	1.5719	1.5837	1.6746

From Table2, the deflections are seen to increase with load as expected. The results have been plotted to show the trend of deflection with number of coefficients (Figure 3), and to show the trend of deflection with load (Figure 4).

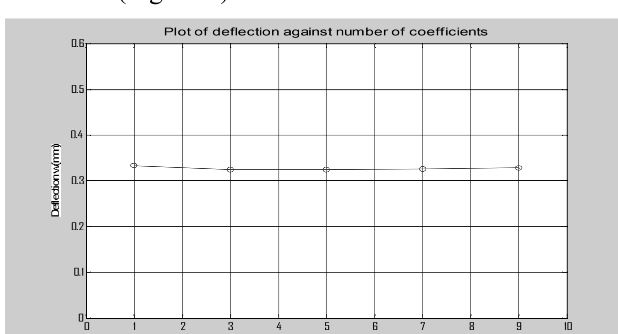

Figure 1: Plot of exact deflections against number of terms

The plot of deflection against number of coefficients or terms (using results of 1N load) in Figure 3 above shows the following:

a) The value of deflection is highest for one term (2.84% higher than the lowest),
b) The value is lowest at three terms (2.84% lower than value at one term),
c) The value at five terms is 2.69% lower than the highest but 0.15% higher than the value three terms, which is the lowest,
d) The value at seven terms is 2.42% lower than the highest but 0.27% higher than the value at five terms that precedes it.
e) The value at nine terms is 1.6% lower than the highest value but 0.8% higher than the value at seven terms.

The plot further indicates that deflections fall steeply when the number of terms is increased from one to three. The deflections thereon increase with number of terms albeit at low gradient. The fall in the value of deflection when the number of terms is increased from one to three and the rise there-on is in-keeping with the results obtained by other researchers who used this method to study uniformly loaded plates. However the results of this research show that the deflections continue to rise at low gradient with increasing number of coefficients for at least up to the ninth term. This shows that convergence is not as fast although it could be possible at higher terms not considered in this research.

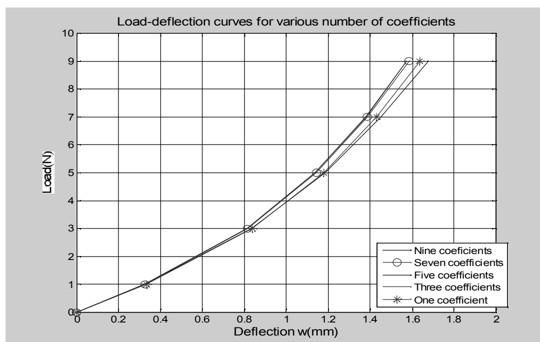

Figure 2: Plot of load against exact deflections

The plot of load against deflection (Figure 4) indicates that deflections increase non-linearly with load. This is in-keeping with the theory of large deflection and compares well with results obtained by Wang and El-Sheikh (2005) for uniformly loaded plates.

4.0 CONCLUSIONS

Large deflection of thin rectangular plates subjected distributed lateral line loads have been mathematically studies using the exact method proposed by Timoshenko. Results obtained have been analysed and discussed leading to the following conclusions;

i. Realistic results can be got using relatively simple computer softwares such to-solve the final load-deflection polynomials hence simplifying the solution significantly.
ii. Although mathematical computer softwares are available to solve load-deflection equations, it is still difficult to get realistic solutions for higher number of coefficients. Good engineering judgment is needed to ascertain accuracy of software output.

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