Simulating Bearing Capacity Failure of Surface Loading on Sand Using COMSOL

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ABSTRACT
Models and formulas for foundation design have been developed and modified over time. The most commonly used formula is that proposed by Terzaghi along with its modifications made by Vesic, Meyerhof and Hansen. However validation tests carried out in Makerere University and University of Cape Town show that Terzaghi's model does not accurately simulate the behaviour of soils as they undergo bearing capacity failure. Because of these findings it is now necessary to develop a model that can simulate the actual behaviour of soils as they experience bearing capacity failure. This will help engineers to generate more economical designs and better assessments of the impacts of foundation loads on the surrounding environment. This study covered the development of a computer model to simulate the behaviour of Philippi Dune sand as it experienced shear failure in the validation test done by Nishaat. The sand was assumed to be an elasto-plastic material that yielded according to the Drucker Prager criterion and finite elements method analysis was done using COMSOL multiphysics. The model developed was able to estimate the failure load and describe the failure mechanism of the Philippi Dune sand in terms of the development of plastic deformation. From the model's failure mechanism it was observed that Terzaghi's model didn't adequately predict the failure mechanism in Phillipi Dune sand.

Keywords: Bearing capacity failure, Numerical model, COMSOL, Terzaghi

1.0 INTRODUCTION

The increasing economic developments all over the world have resulted into the need for taller and bigger structures that will provide more space for offices, shops, accommodation and the like. These structures by the virtue of their sizes and application will be transferring huge loads to the supporting ground below. One of the safety requirements for these structures is an adequately functioning foundation under the prevailing conditions.

Nishaat (2009) investigated the accuracy of Terzaghi's model using a Zwick machine to load soil in a box (Figure 1) that was built in a geotechnical laboratory. The box's dimensions were 1000x150x500mm and the model footings of rigid steel had dimensions of 100x148x16mm and 50x148x16mm. Two tests were carried out on Philippi Dune sand using both steel plates and one test on the Klipheuwel sand using the bigger plate. The angles made by the shear planes to the horizontal and the extent of the failure planes were observed from the box.

It was found that Terzaghi and Meyerhof's models didn't resemble the failure surfaces observed during the laboratory tests (Figure 2). The ratio of observed over theoretical areas of influence varied with footing sizes and soil type. Terzaghi's bearing capacity equation using the residual shear parameters gave the closest value for the bearing capacity of Philippi Dune sand measured from the pressure-settlement curves of the loading tests.

However the use of a physical model in the investigation limited how much Nishaat could observe as regards to the stress distributions, the failure mechanism and the exact extent of plastic deformation. To overcome this limitation, in this study a numerical model was developed so as to simulate the bearing capacity failure in sands and provide data on the distribution of these properties.

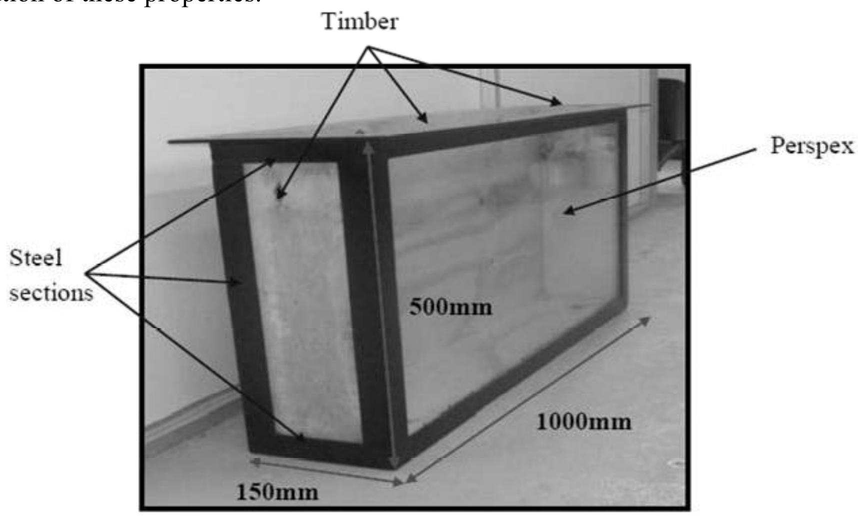

Figure 1: Loading box (Adapted from Nishaat 2009)

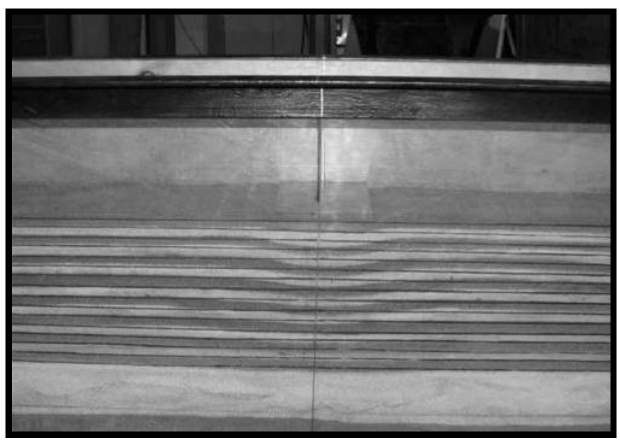

Figure 2: Shear failure in Philippi Dune sand (Adapted from Nishaat 2009)

To develop and calibrate a suitable numerical model the following objectives were carried out:

i) Analysis of the existing data on the bearing capacity failure of Philippi dune sand from the validation test carried out by Nishaat (2009),
ii) Use of COMSOL multiphysics to build a computer model to simulate the bearing capacity failure of the Philippi Dune sand used in the validation test carried out by Nishaat (2009),

2.0 DEVELOPMENT OF THE MODEL

The physical model used in the validation tests carried out by Nishaat was chosen to provide the necessary parameters required to build the simulation in COMSOL multiphysics. The physical model comprised of the following; a loading box (Figure 1), steel plates as model footings and a Zwick Universal tensile and compression machine to apply a vertical load.

It was assumed that the loading box used in the validation test was rigid enough to ensure that the displacements normal to the front and the rear faces were negligible hence plane strain conditions apply. The dimensions of the model's geometry (Figure 3) were the same as those of the loading box and it was constructed in a 2D space.

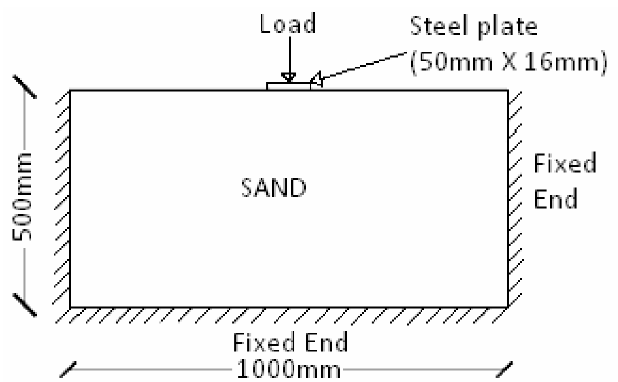

Figure 3: Dimensions, load and boundary conditions used in COMSOL

Boundary conditions: The sides and the base of the model were considered to be fixed ends because the loading box used in laboratory by Nishaat consisted of plywood boards reinforced by steel angles that provided rough and rigid surfaces. The load was applied as a uniformly distributed force on the steel plate. To simplify the model the friction between the steel plate and the sand was assumed to be negligible and soil weightless.

Material properties: The properties of Philippi Dune sand tested with the 100mm long steel plate were used in the model to define the sand. They included:

i) Peak angle of internal friction ($\varnothing_{peak}$) of 34°
ii) Cohesion (c) of 6.7 KPa,
iii) Density (ρ) of 1.713 Mg/m^3,
iv) Poisson's ratio of 0.3.
v) Modulus of elasticity of 7.82e6 Pa.

3.0 NUMERICAL MODEL

Constitutive model: For simplicity the soil was assumed to behave as an elastic perfectly plastic material that failed according to the Drucker-Prager yield criterion. This yield criterion was chosen because it is a function of the mean stress on which yielding in frictional materials is dependent. The yield criterion was given by (Yu, 2006):

$$\mathbf{F} = 3\alpha\sigma_m + \sigma_{eqv} - \mathbf{K} \tag{1}$$

Where the mean stress σ_m was defined by:

$$\sigma_m = \frac{1}{3}(\sigma_x + \sigma_y + \sigma_z) \tag{2}$$

The equivalent deviatoric stress σ_{eqv} was given by:

$$\sigma_{eqv} = \sqrt{\frac{1}{2}[S_x^2 + S_y^2 + S_z^2] + S_{xy}^2 + S_{yz}^2 + S_{zx}^2} \tag{3}$$

Where;

$$S_x = \sigma_x - \sigma_m, \quad S_y = \sigma_y - \sigma_m, \quad S_z = \sigma_z - \sigma_m$$
$$S_{xy} = \tau_{xy}, \quad S_{yz} = \tau_{yz}, \quad S_{zx} = \tau_{zx}$$

And the parameters α and K were given by:

$$\alpha = \tan\phi/\sqrt{((9 + 12\tan^2\phi))} \tag{4}$$

$$K = 3c/\sqrt{((9 + 12\tan^2\phi))} \tag{5}$$

Analysis: the geometry was partitioned into triangular elements using a mesh generator in the program and the solution to the problem was computed.

4.0 FINDINGS

Analysis of the data from the laboratory tests carried out by Nishaat provided only three parameters out of the five required by the model that is; density of 1.713Mg/m^3, cohesion of 6.7 KPa and the angle of internal friction of 34^o. The modulus of elasticity of 7.82e6 Pa and Poisson's ratio of 0.3 were not readily available and were obtained through estimation.

The model built in COMSOL simulated the bearing capacity failure of Philippi Dune sand by showing the development of plastic deformation below the footing. The region of soil that experienced plastic deformation below the footing in the simulation (Figure 4) had an elliptical shape similar to that observed in the validation test (Figure 5) and the maximum traction force below the edge of the footing at the point of failure was similar to the bearing capacity measured in the laboratory test carried out by Nishaat (Figure 6).

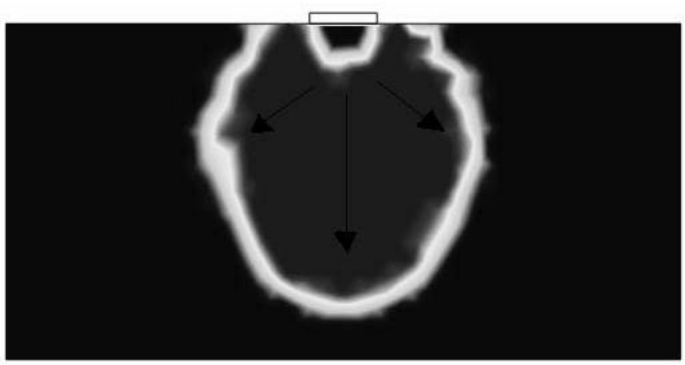

Figure 4: Development of the elliptical bulb

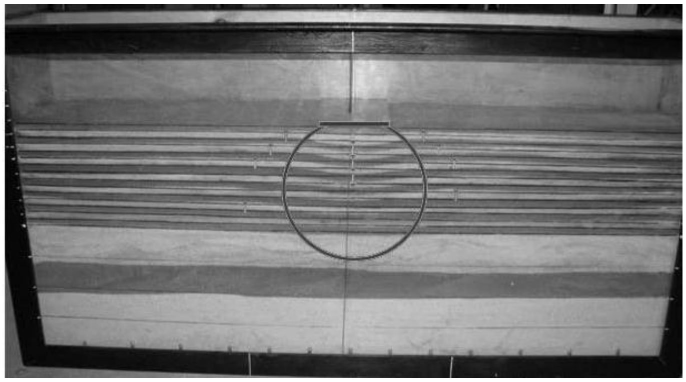

Figure 5: Region of plastic deformation on Philippi Dune sand (Nishaat 2009)

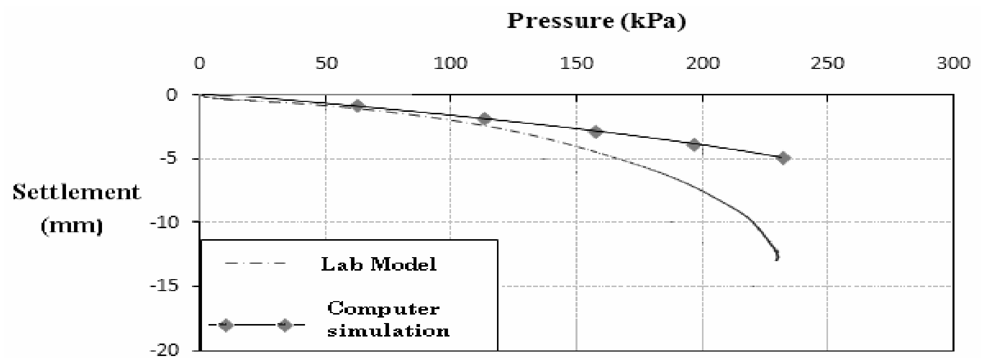

Figure 6: Graph of applied Pressure against the Plate settlement

A further study of the simulation revealed a mechanism of failure in the soil that differed from Terzaghi's bearing capacity theory. The failure was progressive and plastic deformation started at the edges of the footing (Figure 7) and progressed down along lines that were at approximately $45 + {}^{\varphi}/_{2}$ to the base of the footing. This formed a triangular region (Figure 8) that remained in elastic state similar to zone 1 in Terzaghi's model.

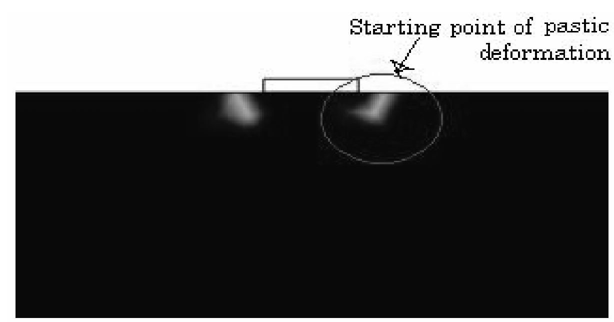

Figure 7: Plastic deformation at the edges of the footing

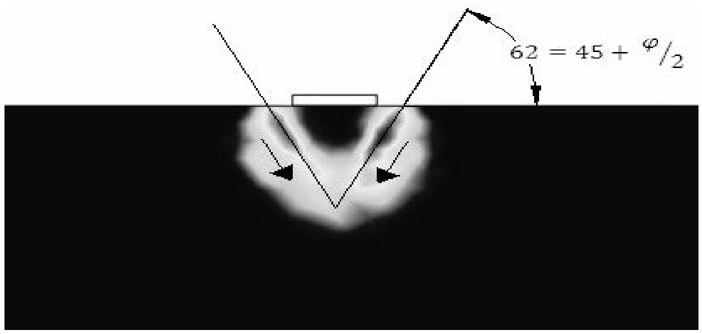

Figure 8: Triangular region in elastic state

The edges of this triangle then expanded outwards into the soil forming a bulb that was an approximate ellipse (Figure 4). This bulb then expanded with increased loading until the point of failure. According to the observations made in the laboratory tests carried out by Nishaat the plastic flow occurs as illustrated in figure 9. According to Terzaghi's bearing capacity model if a weightless soil with cohesion and friction experienced bearing capacity failure. The plastic flow would be over the composite surface abcb'a' (Figure 10) and there would exist three zones in plastic equilibrium. Zone I in elastic state, zone II and Zone III characterised by different shear patterns (Terzaghi & Peck, 1967). However this prediction by Terzaghi's model does not match the observations made in the simulation where the sides of Zone I expanded outwards forming an elliptical bulb.

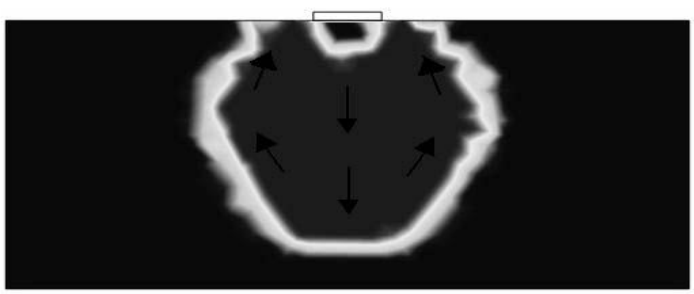

Figure 9: The plastic flow

This variation could possibly be the result of using the Drager-Prager yield criterion.

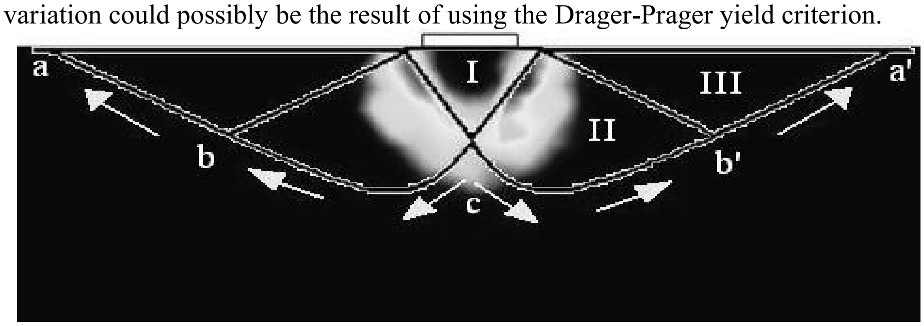

Figure 10: Plastic flow and boundaries of the plastic equilibrium zones

5.0 CONCLUSION AND RECOMMENDATION

From the findings it can be concluded that:

- Analysis of the data from the validation test done by Nishaat illustrated how Philippi dune sand failed in bearing capacity and provided the values of the density, cohesion and the angle of internal friction. The modulus of elasticity and Poisson's ratio were not available and were obtained through estimation.

- The developed model simulated the bearing capacity failure in Phillipi dune sand by showing the region of plastic deformation below the footing and the maximum applied force at the point of failure.
- The failure mechanism of the developed model shows that Terzaghi's theory doesn't adequately predict the failure mechanism in Phillipi dune sand.

Basing on the tests and procedures that were carried out in the study, it is recommended that tests should be done to provide the actual modulus of elasticity and Poisson's ratio of the Philippi dune sand that was used in the validation.

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